Traveling the Interplanetary Superhighway

An Autonomous Spacecraft Navigation System

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1. Executive Summary

This project envisions a new type of spacecraft. Imagine a spacecraft that can navigate throughout the solar system without human intervention. The spacecraft would be small, no more than a few cubic feet, containing equipment for gathering data and sending information back to Earth as well as a form of continuous propulsion, such as an ion drive or a solar sail. This spacecraft would be unique in its ability to roam the solar system autonomously for decades.

Such a spacecraft is possible using the posited Interplanetary Superhighway, the collection of low-energy orbits that would provide fuel-less transport for spacecraft throughout the solar system. Low-energy space travel is similar in many ways to the first explorers traveling the oceans. Sailing ships use winds and currents, natural energy sources on Earth, to travel our oceans. In the same way, spacecraft can use the gravity and movement of planets, natural energy sources in space, to travel our solar system. Maneuvers that utilize gravity in this way are referred to as low-energy orbits.

While scientists agree that this ‘sailing ship of the solar system’ is theoretically possible, actually describing how it would work has proved to be a challenge. Low-energy orbits that reach other planets seem to take prohibitively long amounts of time to fly and are extremely complicated to plan. This project seeks to find efficient types of low-energy orbits and determine how a spacecraft would fly them.

A prototype software system has been developed that would allow a spacecraft to calculate and fly low-energy orbits. Results show that a spacecraft could use minute propulsion capabilities to reach other planets far more quickly than previously thought using a low-energy orbit. This research may represent a practical step forward, transforming the Interplanetary Superhighway from theory to a concrete method for space exploration.

2. Research

2.1 Foundational Literature

There are several major types of low energy orbit planning, each of which focuses on navigating a different area of the solar system. The first type investigates navigating the area near Lagrange points and uses differential equations to describe the “manifolds” created by the paths of un-propelled particles as they move through the region and are affected by gravity. Lo (1), Koon, Marsden, and Ross (2) have contributed greatly to this area. This type of low energy orbit planning is most closely related to this project as it focuses on the same area of space and utilizes similar orbit shapes. In this project, these orbits have been used to navigate near the Earth and then enter interplanetary space. Another type of low-energy orbit planning focuses on systems with multiple bodies in resonant orbits. This approach creates a “kick function” to characterize how much a planetary body will affect another objects’ orbit depending on how closely they pass each other. Although this works best in the system of Jupiter’s moons, where orbit periods are relatively small and there are many moons with which to interact, the studies of Grover and Ross (3) reveal principles that are useful when low energy spacecraft initially enter interplanetary space. Their method may be used to adjust the orbit of a spacecraft so that it can reach another planet. The third type of low-energy orbit planning focuses on navigating interplanetary space using gravity assist maneuvers with several planets. Studies in this area have achieved fascinating results, and show that it is possible to navigate the entire solar system through a network of gravity assist maneuvers at different energy levels. Strange and Longuski demonstrate the general principles (4), and Petropoulos, Longuski, and Bonfiglio show how to optimize these gravity assists (5). Since navigation with gravity assists allows for relatively easy, low energy access to the solar system once a spacecraft can get to another planet, this project focuses on planning the low-energy orbits near Earth and connecting these orbits to the larger interplanetary gravity-assist network.

2.2 Terms

Earth's Neighborhood- is the region near Earth including the Moon, the Earth-Moon Lagrange points, and the Earth-Sun Lagrange points.

Forbidden region- describes the area in a two body system that an object cannot reach given a limited amount of energy. The forbidden region, also referred to as a Hill's region, restricts the motion of low-energy object. See (2) for a mathematical definition.

Heteroclinic connection- shown in Figure 1, a low-energy maneuver that allows a spacecraft to exit a gravity well for reduced energy.

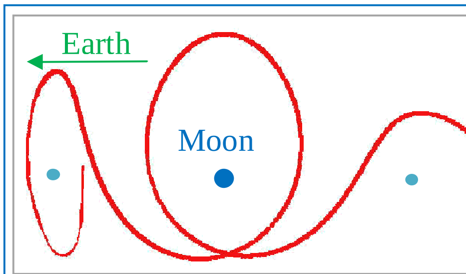


Figure 1: Heteroclinic Connection
This path was a major focus of the work of Martin Lo and Shane Ross (1), seminal researchers in the field of traditional low-energy orbits. In this maneuver, the smaller body, the Moon in this case, is used to help pull spacecraft from the L_1 point to the L_2 point. Since L_1 is inside Earth's gravity well and L_2 is outside, this effectively allows spacecraft to exit for reduced energy.

Interplanetary Superhighway (IPS)- the use of low-energy paths to move throughout the solar system. As described in Ross and Lo's paper (6), the "Interplanetary Superhighway System (IPS) provides ultra-low energy transport throughout the Earth's Neighborhood." This project extends the range of the Interplanetary Superhighway, describing low-energy paths that can leave the Earth's Neighborhood and reach other planets by combining several fields of low-energy orbit research.

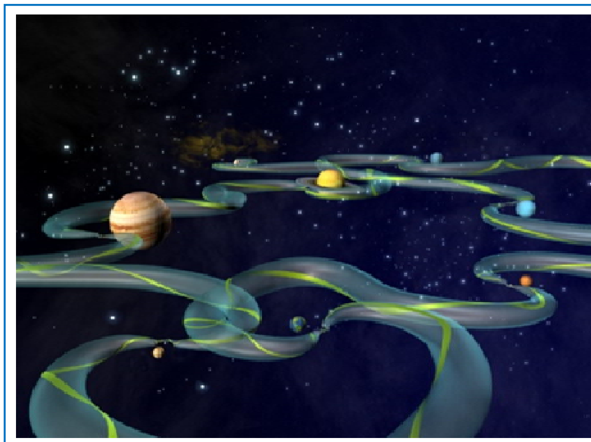


Figure 2: the Interplanetary Superhighway
At right is an artist's conception of what the IPS might look like.

Jacobi integral- relates position, velocity, and total energy of an object within a two body system. This system has the larger body with mass M, at $(-m, 0)$ and the smaller with mass m at $(M, 0)$. A spacecraft of mass 1 at position (x, y) and with velocity $\dot{x}$ and $\dot{y}$. R is the distance of the spacecraft from the larger body, and r is the distance from the smaller body. n is the mean motion, or 2π over the orbital period For some constant energy, $C_j = n^2(x^2 + y^2) + 2\left(\frac{M}{R} + \frac{m}{r}\right) - (\dot{x} + \dot{y})$. In other words, the energy equals the centripetal energy, the gravitational energy, and the kinetic energy. For a complete mathematical definition, see Szebehely (7). This project's software system uses the Jacobi integral to set initial positions and velocities of spacecraft in a two body system.

Lagrange Points- five points of equilibrium in a two body system. L_1 and L_2 are most prominent in this study. In a two-body system of planets masses m and M with distance R between them, L_1 and L_2 lie a distance of $R\left(\sqrt[3]{\frac{m}{3M}}\right)$ on the inside and outside of the smaller body on the small body- large body line. L_1 and L_2 are unstable, meaning that objects will diverge from their positions quickly near these points, making them useful for navigation.

Low-energy orbit- the path an object takes as it is effected by the forces planetary bodies exert without the use of self-propulsion, or ΔV. The movement of these objects is especially sensitive to the gravity of planets and the divergence of the nearby space (referred to as the Lyapunov Exponent), making these paths complex and usually non-elliptical. In this field, 'orbit' designates any spacecraft path, regardless of shape.

Lyapunov Orbit- a path around an L_1 or L_2 point that relies on both the gravitational attraction from the two bodies and the coriolis effect. Lyapunov orbits are the 2-dimensional version of "halo orbits," and have a characteristic "bean shape." Essentially, this orbit utilizes the navigational properties of the unstable Lagrange points to create a holding pattern, allowing planetary bodies to move to more favorable positions for subsequent maneuvers.

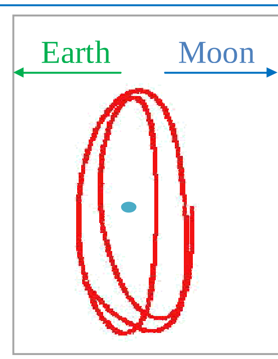


Figure 3: Lyapunov Orbit
A holding pattern.

Station Keeping- the process where a spacecraft uses small positioning maneuvers to maintain its orbit. Low-energy orbits are extremely sensitive to initial conditions and perturbations. This means that any spacecraft flying a low-energy orbit will need to use station keeping methods to correct for the inevitable navigation imprecision.

An ion drive is a good choice for low-energy orbit station-keeping. Ion drives accelerate ions in a propellant such as xenon gas to extremely high speeds, creating relatively high thrust for very little fuel. This means that an ion drive can provide *continuous propulsion*, extremely small but continuous thrust, for decades without needing to be re-fueled. A spacecraft would use this thrust to perform station keeping maneuvers and successfully fly sensitive low-energy paths.

Synodic CM (center of mass) coordinate system- When a spacecraft is near two planetary bodies, such as the Earth and the Moon, their gravity dominates the spacecraft's movement. Therefore, it is often helpful to look at movement relative to those two bodies by drawing the two bodies in the same place consistently. This frame of reference is called a synodic coordinate system and can be a useful tool for understanding spacecraft movement. Pictures drawn in synodic systems will have planets labeled in the picture. This is why low-energy orbits do not always look elliptical.

Figure 4: A common synodic system

2.3 Applications of Low-Energy Orbits

There are several instances where spacecraft have utilized low-energy orbits to great effect. The Genesis spacecraft used a series of Earth-Sun L_1 and L_2 Halo orbits to collect solar wind samples (1) (9). Another example is the Hiten, a Japanese spacecraft designed to relay signals for the Hagoromo. After the Hagoromo failed, a low-energy transfer to the moon was executed, allowing the Hiten to gain moon orbit even though it had 10% less fuel than was believed to have been needed (10), making the mission a success. The most famous use of low-energy maneuvers are the gravity assists used by Mariner 10, Pioneer 10 and 11, and Voyager I and II.

Low-energy orbits allow spacecraft to navigate in Earth's Neighborhood for extended periods of time, requiring only enough fuel to correct for navigation error. One application of these paths currently being discussed could aid in climate engineering and mitigate global warming. Many unmanned spacecraft could be launched using these paths, erecting a deflective sun shield at the Earth-Sun L_1 point. Also, Earth-Moon Lagrange points have been suggested as positions where repair crews could gain access to satellites. The possibilities for low-energy paths within the Earth's Neighborhood are significant.

Another practical use of low-energy orbits would be to aid in the transit of heavy spacecraft, such as unmanned supply crafts or cargo crafts for asteroid mining. Since it requires a large amount of fuel to initially launch heavy spacecraft, there would be little fuel still available in space with which to maneuver, making such a mission a good candidate for low-energy orbits. Since these paths take longer to execute than traditional methods, these supply crafts could be launched ahead of the primary mission and then wait in orbit near the destination. This method would allow larger-scale missions in remote parts of the solar system because of the increased availability of materials. In the case of asteroid mining, low-energy orbits could provide a cost-effective method for transportation across vast distances.

3. Novel Method for Identifying Low-Energy Trajectories

This project focuses on developing a software tool that would allow a spacecraft to calculate and fly low-energy orbits. The resulting software automatically finds low-energy paths that follow a specified itinerary, as described by satisfying spatial boundary lines that are set up at critical areas in the simulation region. This approach can find complex orbits throughout different two body systems and automatically adjusts for N-Body perturbations, making it a good candidate for practical orbit planning.

The program was written in its entirety by the researcher in C++, a language with multithreading capabilities and a strong object-oriented nature. The positions of the planets are calculated using on ellipses and accurate to a specific date, as given by the equations of Schlyter (11). The program then uses Runge Kutta 4, as described by Hut, Makino, and Heggie (12), to track the positions of "tracer particles" as they are affected by the gravity and movement of the planets. Alternatively, the program converts tracers that enter interplanetary space into ellipses so that they can be simulated for longer periods of time. The user interface consists of a browser displaying various views of the simulation region as well as several data export files.

Note that like most research on low-energy orbits, the simulation in this program is two dimensional. This is because a three dimensional simulation adds to the compute intensity, but does not fundamentally change the problem in any way. The same methods with very slight modifications may be used in conjunction with a more complete simulation engine should a more precise orbit be required. The current setup allows for realistic planet positions and high-accuracy numerical integration, which has previously been investigated.

3.1 Algorithm: Itinerary-Based Optimization

Low-energy orbits are most interesting in areas of space that are highly chaotic, meaning that completely different paths can start with the same velocity mere centimeters apart. Their sensitivity necessitates the use of an automatic computer system to pinpoint these paths. This project develops a guided optimization algorithm to find specific paths by simulating many tracers, grading each tracer's path based on how well it followed a specified itinerary, and then simulating more tracers like the one that best satisfied the itinerary. This may

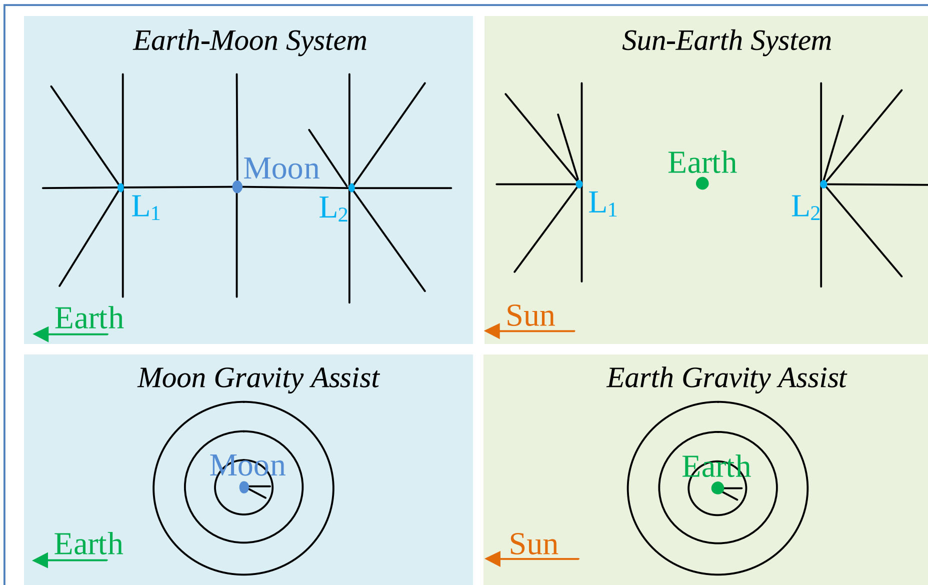


Figure 5: Boundary Positions Near Earth
Synodic, CM system. For left picture, Moon-centric with L_1 on left. For right picture, Earth-centric with L_1 on left. This is the layout of the boundaries which are used to analyze the paths of the tracers. Each boundary is assigned a number, and itineraries may be specified by a list of numbers in order, representing the order in which the tracers must pass over the boundaries.

be viewed as a genetic algorithm, where the fitness of each tracer is determined by its path's grade. The simulation also automatically backs up and introduces new tracer positions when it detects that more of the itinerary is not being achieved, which is similar to the 'genetic mutations' in genetic algorithms.

Figure 5 depicts the primary boundary lines used to construct itineraries. These lines are supplemented by circular boundaries around planets, which detect when spacecraft go near planets, as well as other maneuver-specific boundaries.

An itinerary is described as a list of spatial boundaries each tracer must cross in order. The objective of the program is to find a trajectory that satisfies the given itinerary. The program tracks the boundary crossings of each tracer so that it can refine on tracer trajectories that yield the most correct list of boundary crossings, as shown in *Figure 6*. By continuously refining the initial positions to fit the requirements for a long itinerary, the program may find an appropriate path.

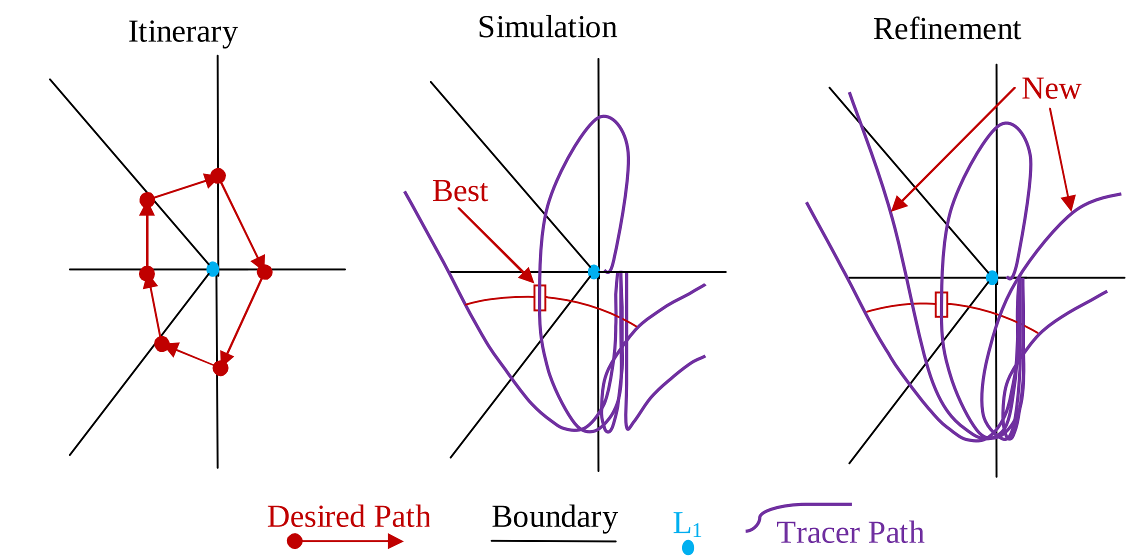


Figure 6: Optimization of Spacecraft Trajectories
Synodic. The program identifies the "best" tracer or tracers as well as the range where the correct path may lie. Then it generates new tracer positions in this area and simulates again.

The itinerary is given to the program as a string and then parsed. There are various identifiers that may be included to help the computer complete the itinerary. Boundaries may be specified as optional when it is unclear if they are necessary. The user can also specify a boundary to "reload" on; when the program detects that all the tracers have successfully completed the itinerary up to that point it will save the positions of the tracers as they cross the boundary and then restart the next simulation from that point, saving simulation time. In the same way, the user can specify to turn on or off sets of boundaries when the tracers complete a portion of the itinerary, such as turning on gravity assist boundaries at the appropriate moment. If the program detects that it is not making progress completing more of the itinerary it will back up to a previous state, introduce more tracers, and continue. It can also save the positions of tracers to a file and then start a simulation with this stored data.

3.2 Autonomous Spacecraft Navigation System

Any low-energy orbit spacecraft must have some way of performing station keeping maneuvers in order to fly these highly sensitive paths. An ion drive is a good choice for a type of continuous propulsion that would provide the minute amounts of thrust required. However, there may also be times when the spacecraft does not need to perform station keeping. In these instances, there might be a way to put the continuous propulsion to good use in raising the spacecraft's energy. The autonomous spacecraft navigation system acts as the navigation computer for a spacecraft, calculating how to use continuous propulsion for both station keeping as well as increasing energy. It works with the following steps:

First, the spacecraft uses its onboard computer to project where it will be some time interval in the future, perhaps 30 minutes. Next, the software uses the itinerary-based algorithm to calculate how the spacecraft could best use its continuous propulsion to fly the desired route. All of the computer's initial guesses are located at the spacecraft's anticipated position and differ by the angle they point their continuous propulsion. The program finds the optimal angle for the continuous propulsion that would allow the spacecraft to fly the correct route. At the end of the time interval, the spacecraft arrives at the anticipated location and uses the direction of continuous propulsion it just calculated for the next 30 minute period. Meanwhile, it again projects its position forward and re-calculates its propulsion angle.

This allows the spacecraft to perform station keeping by updating its continuous propulsion angle to adjust for navigational errors.

This system represents a significant change in the way spacecraft operate. Not only would a spacecraft using this software system be able to fly low-energy orbits, it would also be able to calculate its own movement. Current spacecraft rely on mission control to instruct them on how to maneuver. This autonomous navigation system allows spacecraft to independently calculate and fly low-energy orbits.

3.3 Multithreaded Code

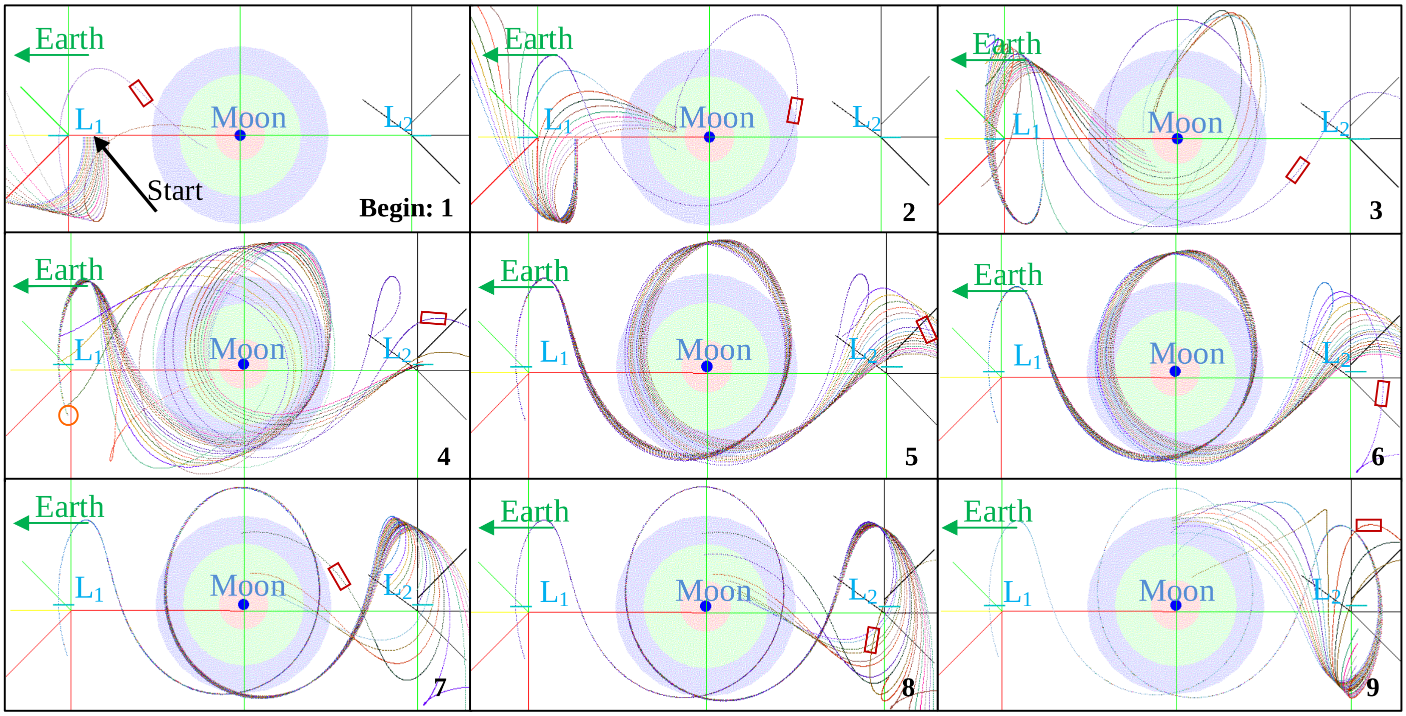
In order to simulate more complex and accurate maneuvers the program has been adapted for use on multi-core processors. From 1 to 4 cores, the speedup is about 3, but this speedup changes greatly depending on conditions in the simulation.

4. Results

4.1 Maneuvering in the Earth's Neighborhood

There are several orbit patterns that allow for maneuvering around the Earth's Neighborhood. First, a heteroclinic connection is used to exit the Earth's gravity well. A tracer may move from a heteroclinic connection directly into interaction with SE Lagrange points. Lyapunov orbits may be used to orbit either EM or SE Lagrange points as a holding pattern. Using these maneuvers, it is possible for the algorithm to navigate from the inside of Earth's gravity well to any of the EM or SE Lagrange points with relative ease.

Figure 7 is an example of such navigation. The tracers begin orbiting the EM L_1 point with the smallest amount of energy possible to exit the gravity well. They perform a heteroclinic connection, a EM L_2 Lyapunov orbit, then transfer to the SE Lagrange points, performing two SE L_1 Lyapunov orbits. Extensive testing has shown that similar maneuvers can navigate throughout the area.



Frame of reference change: the program has finished the heteroclinic connection and Lyapunov orbit in the Earth-Moon system, and now focuses on finding the Lyapunov orbit in the Earth-Sun system.

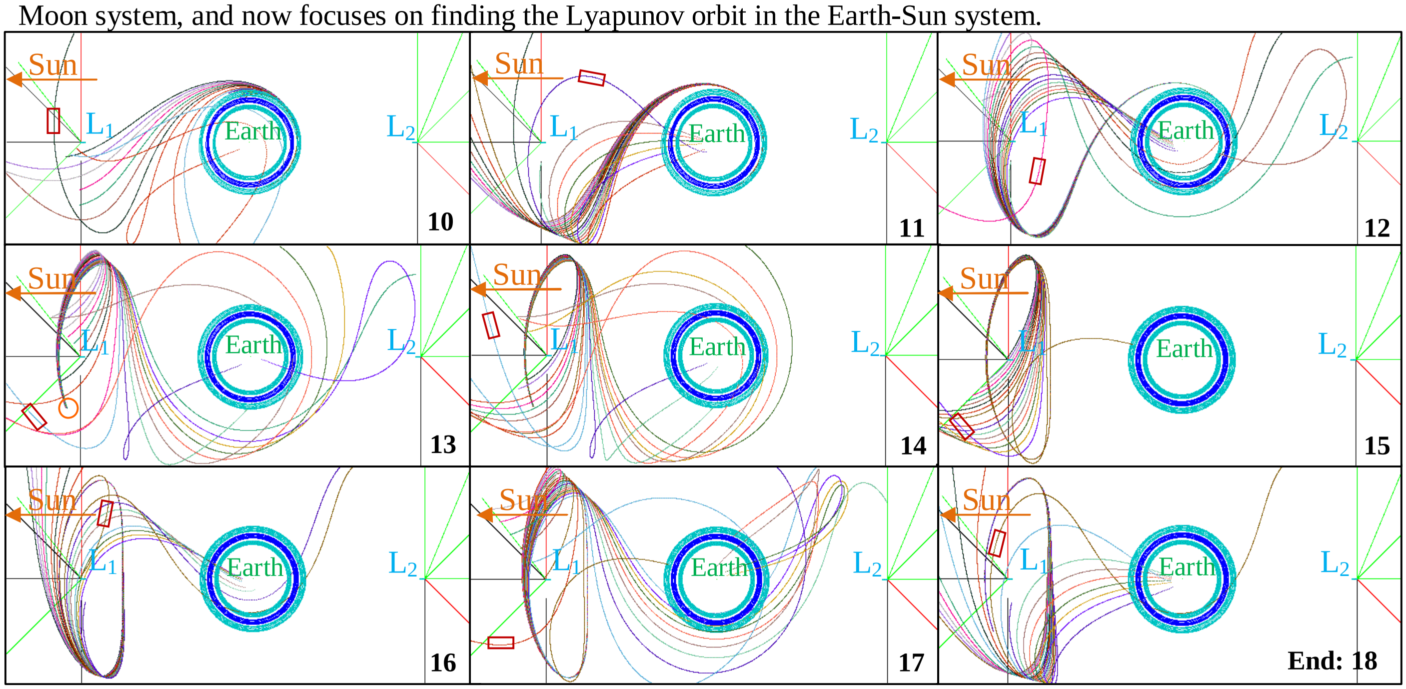


Figure 7: Navigating the Earth's Neighborhood

Synodic, CM system. For top pictures, Moon-centric with L1 on left. For bottom pictures, Earth-centric with L1 on left. ▯ indicates the tracer with the best grade. ○ indicates a re-loading after previous portions of the itinerary have been successfully completed. The black numbers at the bottom right corner show the order of the pictures, each showing one iteration. This shows the program finding a heteroclinic connection followed by a EM L_2 Lyapunov orbit and then two ES L_1 Lyapunov orbits.

4.2 Strategies for Diverging from Earth's Orbit

Reaching another planet with the minimum amount of energy needed to exit Earth's gravity well, and without further propulsion, is clearly a challenge. However, this project's approach allows N-body effects to be taken into account, which makes such movement possible. In this case, we will focus on creating eccentric orbits to reach either Venus or Mars. As spacecraft exit the Earth's Neighborhood for the first time, they have the unique opportunity to interact with the Earth-Moon system in such a way that they diverge from Earth's orbit as much as possible. 'Exit strategies,' shown in *Figure 8*, are analyzed in this section.

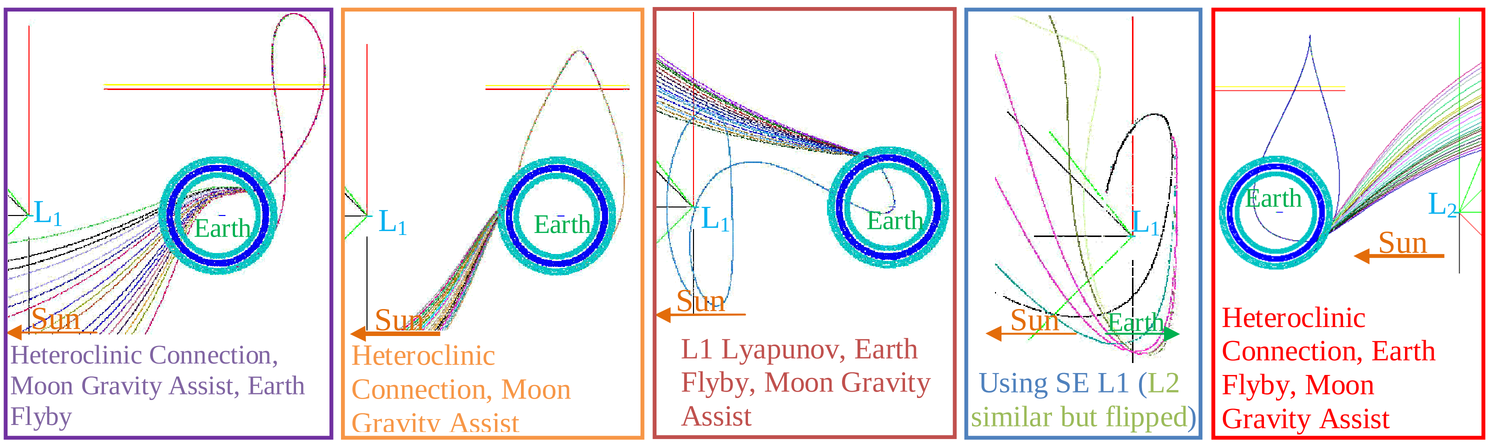


Figure 8: Exit Strategies
Synodic, CM system. SE system with L_1 on left. The pictures show examples of each type of exit strategy.

It should be possible to diverge more from Earth's orbit with the help of the Moon than would normally be possible with just the Earth and the Sun. To test this, the researcher chose several exit strategies in which the tracers interact with both the Earth and the Moon during their departure from the region. As the tracers enter interplanetary space, the program calculates and records their orbits around the Sun. The results of these calculations are shown in *Figure 9*. Using an SE Lagrange points to exit the system would be the only method available if the Moon was not present. As shown, one of the methods for interacting with the Moon was not as good as using the Lagrange points, while two of them were better.

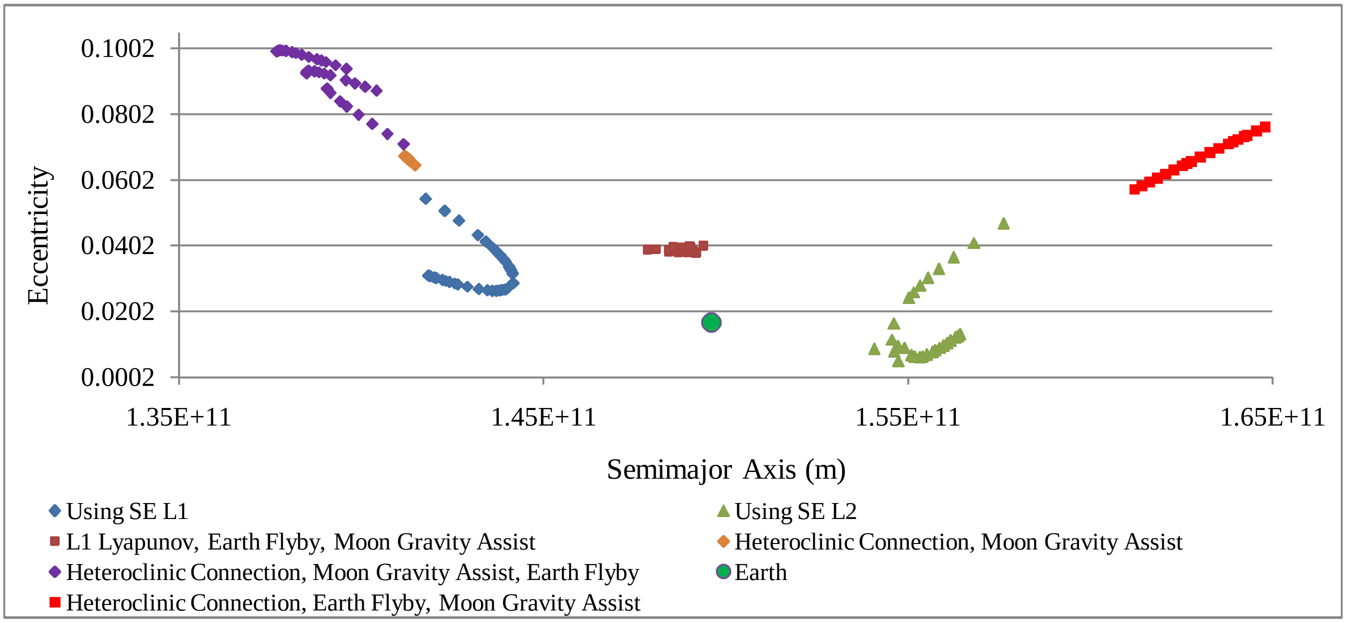


Figure 9: Comparison of Methods for Diverging from Earth's Orbit: Resulting Semimajor Axis and Eccentricity
The graph shows the resulting orbits for each of the tested exit strategies relative to Earth's Orbit.

The goal is for the tracer's orbits' closest point to the Sun (perigee) to be closer than Venus' farthest point from the Sun (apogee), or have the tracer's apogee be farther than Mars' perigee. The difference between perigee and apogee for Venus are plotted in *Figure 10*, and show the relative merits of each exit strategy.

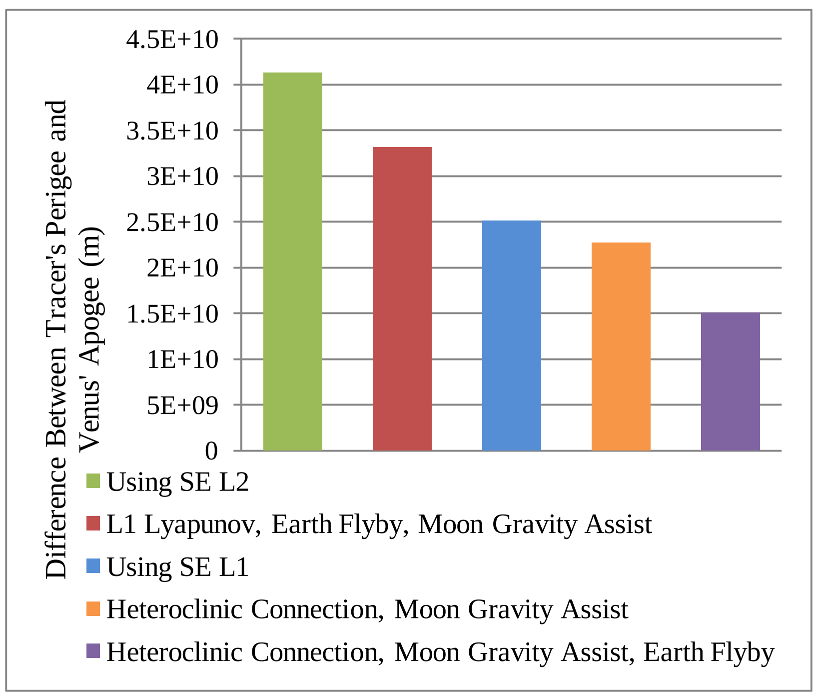


Figure 10: Effectiveness of Each Exit Strategy
Exit strategies with the smallest possible distance are most effective, showing the "Heteroclinic Connection, Moon Gravity Assist, Earth Flyby" method to be best.

As shown, exit strategies that took advantage of the Moon produced highly eccentric orbits with lower semi major axis and are closest to being able to interact with Venus, at which point they could use gravity assists to reach the rest of the solar system. Similarly, the "Heteroclinic Connection, Earth Flyby, Moon Gravity Assist" method is closest to being able to interact with Mars.

4.3 Increasing Energy with Continuous Propulsion

The simulation-based approach allows for extra perturbations to be easily taken into account, making spacecraft with small continuous thrust a natural extension of the current research. Near Earth, continuous propulsion can be used to increase the energy of a spacecraft so that the forbidden region disappears. While it is possible to travel the solar system without it, small amounts of continuous prolusion have dramatic effects on the time required to reach other planets using low-energy trajectories.

Since any spacecraft flying a low-energy orbit would need to perform station keeping it must have some form of propulsion. However, when this thrust is not being used for station keeping it should be used for something else. Near Earth it is a good idea to use continuous propulsion to increase the energy of the spacecraft, allowing a spacecraft to perform more complex maneuvers more quickly. This is done by pointing the thrust in the proper direction to increase the velocity of the spacecraft in the synodic system. This is called *synodic acceleration,* as shown in *Figure 11*. In Interplanetary Space, continuous propulsion can be used to change the orbit of a spacecraft so that it reaches another planet.

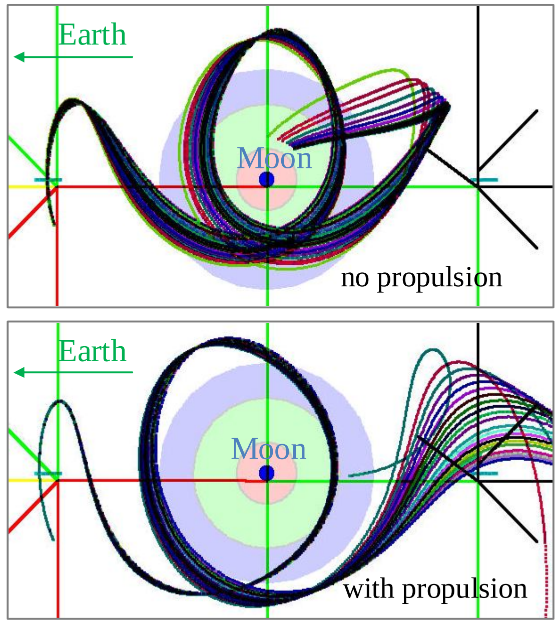


Figure 11: Using Continuous Propulsion to Increase Energy
The spacecraft in the top pictureare attempting to perform a heteroclinic connection but do not have enough energy. Adding a small amount of synodic acceleration allows these spacecraft to successfully complete the maneuver. Note that this is far more continuous propulsion that is normally used.

This project's simulation based approach puts it in a unique position to study low-energy orbits with continuous propulsion. It has simulated significant orbit changes using parameters for an ion drive that provides an acceleration of only $6.5 * 10^{-5} m/s^2$. The autonomous spacecraft navigation system allows our spacecraft to use this small amount of continuous propulsion for both station keeping and increasing total energy.

4.4 Changing an Orbit in Interplanetary Space with Continuous Propulsion

Once a spacecraft has exited Earth's Neighborhood using an exit strategy it is well on its way to another planet. However, there is very little a spacecraft can do to reach the next planet while it is in interplanetary space. Current low-energy research suggests timing interplanetary orbits so that they are in resonance with the next planet, allowing the spacecraft to get a "kick" off a planet every 10+ years when they pass by each other. This method would take decades to reach another planet, making it impractical.

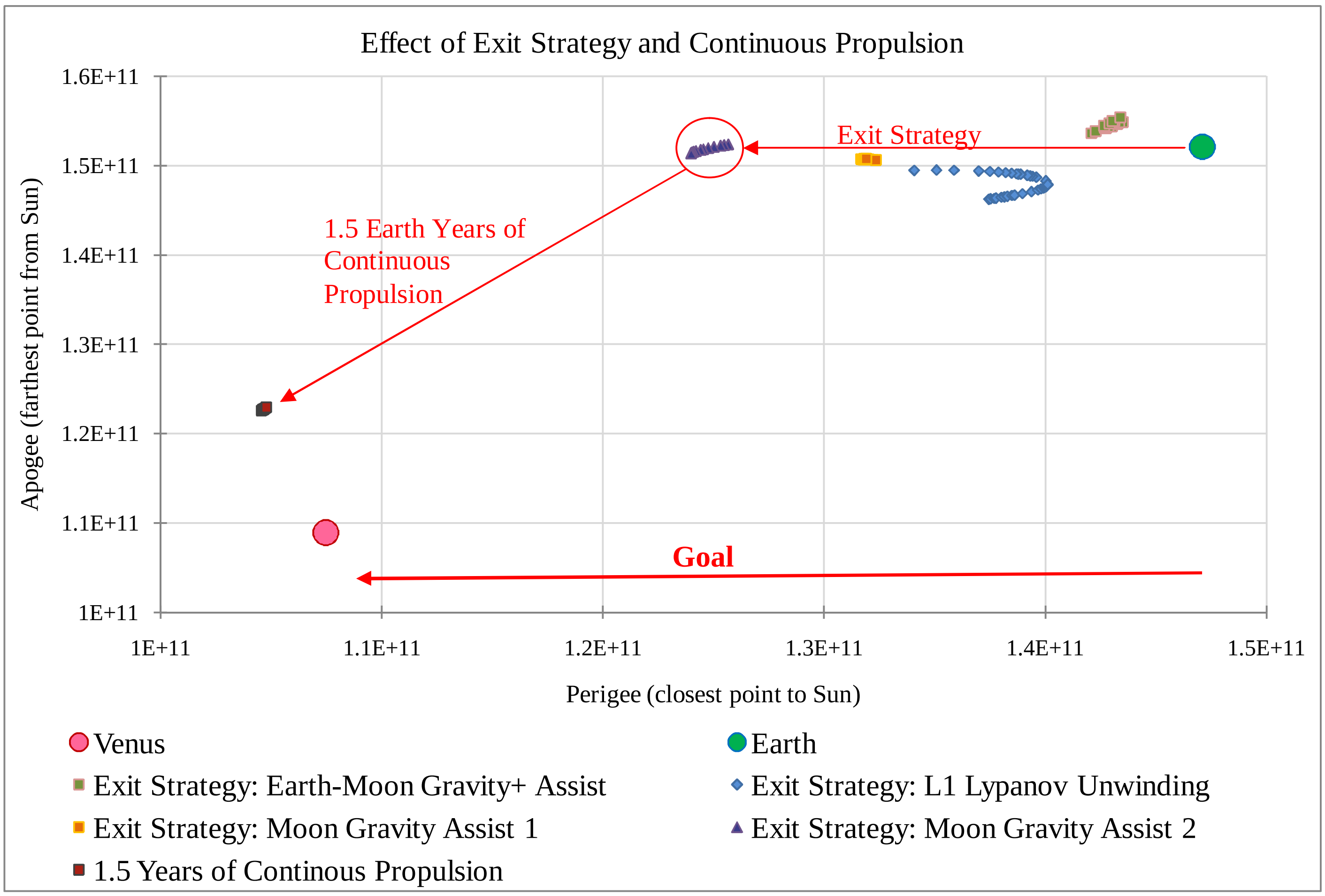


Figure 12: How to Get to Venus
This is a concept diagram showing how it is possible for a spacecraft to do a gravity assist off of Venus using a combination of an exit strategy and tiny amounts of continuous propulsion while in interplanetary space.

Continuous propulsion can be used to greatly decrease the amount of time needed to reach another planet. For example, for a spacecraft to reach Venus from Earth its closest point to the Sun must be closer than Venus's farthest point, as indicated by the goal arrow in *Figure 12*. This can be accomplished in two steps. First, the spacecraft executes an effective exit strategy which throws its orbit far from Earth's as it enters interplanetary space. Second, minimal continuous propulsion may be used to further decrease the spacecraft's apogee, in this case taking only 1.5 years. With proper phasing, this spacecraft could reach Venus using only minute amounts of continuous propulsion and low-energy orbits.

Once a spacecraft reaches another planet, it can perform a gravity assist. One example is the famous VEEGA (Venus-Earth-Earth Gravity-Assist) used by Galileo, and others as described by Petropoulos, Longuski, and Bonfiglio (5) which allow spacecraft to reach the rest of the solar system.

5. Conclusion

A software system has been developed that would allow a spacecraft to autonomously calculate and fly low-energy orbits. The system can navigate throughout Earth's Neighborhood, has been used to test various exit strategies, and incorporates continuous propulsion for station keeping and increasing energy. This project has demonstrated how the 'sailing ship of the solar system' would work both by discovering paths that are time-efficient and by creating a software system that would allow the spacecraft to navigate autonomously.

There are several areas of innovation in this project. First, this approach is a viable option for onboard autonomous spacecraft navigation because it could allow a spacecraft to calculate its path on-the-fly in space and does not require pre-planning of the entire route. Second, the software system has identified several unexpected low-energy orbits that benefit from the effects of many planets, such as those used for diverging from Earth's orbit which utilize the Moon. These new paths have been shown to be significantly favorable to the 2-body alternatives when trying to navigate between planets. Third, this method allows continuous propulsion spacecraft to be researched, a topic that was inaccessible with previous methods. Finally, this method demonstrates the practical connection between the methods of invariant manifolds, resonant transitions, and gravity assists. By taking into account all of the planets it is possible to unify the understanding of low-energy orbits in specific cases with specific goals with their combined, practical application in the N-body solar system. The conceptual Interplanetary Superhighway is concretely illustrated through this approach.

6. Future Work

Current research is focused on finding more explicit examples of possible interplanetary paths for spacecraft. This project will also assess the performance of the program to discover the type of most appropriate type of processor needed for a low-energy spacecraft to safely calculate low-energy orbits. Improving the autonomous spacecraft navigation system for fault tolerance is also a work in progress.

7. Acknowledgements

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